MA 3046 - Matrix Analysis

Problem Set 7 - Section V - Eigenvalues (Partial Set)

1. Normally, we call $\mathbf{x}$ an eigenvector of $\mathbf{A}$ if $\mathbf{A}\,\mathbf{x} = \lambda\mathbf{x}$, i.e. the "input" and "output" are parallel when we multiply $\mathbf{A}$ on the right by a (column) vector.

However, it is also possible to define *left-hand eigenvectors*, i.e.the (column) vector $\mathbf{y}$ is a left-hand eigenvector of $\mathbf{A}$ if, for some scalar η,

$$\mathbf{y}^T\,\mathbf{A} = \eta\,\mathbf{y}^T$$

Show that:

(i) $\mathbf{y}$ is a left-hand eigenvector of $\mathbf{A}$ if and only if $\mathbf{y}$ is also a "normal" (i.e. right-hand) eigenvector of $\mathbf{A}^T$.

(ii) The eigenvalues of $\mathbf{A}$ and $\mathbf{A}^T$ are identical. (Hint: Use properties of the determinant on the respective characteristic polynomials.)

(iii) What, if any, relationship exists between the eigenvectors of $\mathbf{A}$ and those of $\mathbf{A}^T$ when all of the eigenvalues are *distinct*.

2. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 4 & -7 & -5 \\ -2 & 3 & -1 \\ 2 & -5 & -1 \end{bmatrix}$$

Conduct five iterations of the basic power method, starting with $\mathbf{x}^{(0)} = [\ 1\ \ 1\ \ 1\ ]^T$, with normalization (using the infinity norm) after each step, and estimate the dominant eigenvalue and is associated eigenvector. Compare your answer with the exact (MATLAB) answer.

3. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 7 & 21 & -39 \\ 6 & 28 & -42 \\ 3 & 15 & -23 \end{bmatrix}$$

Conduct five iterations of the basic power method, starting with $\mathbf{x}^{(0)} = [\ 1\ \ 1\ \ 1\ ]^T$, with normalization (using the infinity norm) after each step, and estimate the dominant eigenvalue and is associated eigenvector. Compare your answer with the exact (MATLAB) answer.

4. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 10 & -7 & -12 \\ 2 & 1 & -12 \\ 11 & -11 & -3 \end{bmatrix}$$

Conduct three iterations of the basic power method, starting with $\mathbf{x}^{(0)} = [\,1 \;\; 1 \;\; 1\,]^T$, with normalization (using the infinity norm) after each step, and show that, in this instance, the method apparently will never converge. Using the exact eigenvalues and associated eigenvectors of $\mathbf{A}$ as computed by MATLAB, explain what went "wrong" here.